The research and development learning curve 2010-2014: Wits Maths Connect Secondary (WMCS) project

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Professor Jill Adler
FRF Chair of Mathematics Education
and the WMCS project team

School of Education
University of the Witwatersrand
2009 - Call for proposals
Research and Development Chairs in Mathematics Education

• To improve the quality of mathematics teaching at previously disadvantaged secondary schools
• To improve the mathematics results (pass rates and quality of passes) as a result of quality teaching and learning
• To research sustainable and practical solutions to the mathematics crisis
• To develop research capacity in mathematics education
• To provide leadership and increase dialogue around solutions

BRIDGING PRACTICES

Skovsmose – 2008
90% of the research in mathematics education is in service of 10% of the world’s children – typically in resourced environments
Access for all - learning for some


Can a research informed professional development intervention

* Shift this curve?

* Thicken pipeline within the secondary school?
There is compelling evidence that socio-economic status is the strongest predictor of educational success in school (e.g. Coleman et al, 1966; Hoadley, 2010). This, however, does not mean that quality differentials in schooling do not matter. Indeed, recent studies of quality within schools have argued that ‘achievement in countries with very low per capita incomes is more sensitive to the availability of school resources’ (e.g. Gamoran & Long, 2006, p.1. Social justice imperatives thus demand that we investigate what happens in schools and how practices might be changed in order to mediate greater education success of poor learners.
Important results

“the spine”
More learners are obtaining A, B and C-symbols in Grade 12 Mathematics. More careful selection of learners for Mathematics has substantially reduced the numbers scoring below 30%.

<table>
<thead>
<tr>
<th>No. of A, B, C symbols</th>
<th>% A, B, C symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>79 50 74 90</td>
<td>10.9% 8.6% 13.3% 18.4%</td>
</tr>
</tbody>
</table>
Tracking learners

• ICCAMs algebra diagnostic test
• Means to focusing intervention
• Honours and masters studies

Teacher knowledge and identity

• Pre- and post interviews
• Smk/pck scenarios
• identity
Learning gains

Investigating learning gains in relation to teachers’ participation in professional development courses

Intervention group and control group of teachers
Pre- and post-test with 800 Grade 10 learners in 5 project schools over 1 year

Learners taught by teachers who had completed a TM course made **bigger gains** than those taught by teachers who had not participated in a TM course. These learners had a **lower average pre-test score** than the control group but a **higher average post-test score**.
### Teachers' learning - mathematics

<table>
<thead>
<tr>
<th>Course, year</th>
<th>Registered</th>
<th>Completion</th>
<th>Success</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM 1 2012</td>
<td>21</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>TM 1 2013</td>
<td>15</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>TM 2 2012-13</td>
<td>15</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>TM 2 2014</td>
<td>21</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

### Teachers' MDI - mediation of the object of learning

<table>
<thead>
<tr>
<th>Exemplification</th>
<th>Explanation</th>
<th>Engagement</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exemplification</strong></td>
<td><strong>Tasks</strong></td>
<td><strong>Naming</strong></td>
</tr>
<tr>
<td><strong>Examples</strong></td>
<td><strong>2012</strong></td>
<td><strong>2012</strong></td>
</tr>
<tr>
<td><strong>2012</strong></td>
<td><strong>2012</strong></td>
<td><strong>L2-L1</strong></td>
</tr>
<tr>
<td><strong>2013</strong></td>
<td><strong>2013</strong></td>
<td><strong>L2-L1</strong></td>
</tr>
<tr>
<td><strong>T1</strong></td>
<td><strong>L1</strong></td>
<td><strong>L3</strong></td>
</tr>
<tr>
<td><strong>T2</strong></td>
<td><strong>L3</strong></td>
<td><strong>L3</strong></td>
</tr>
<tr>
<td><strong>T3</strong></td>
<td><strong>L3</strong></td>
<td><strong>L3</strong></td>
</tr>
<tr>
<td><strong>T4</strong></td>
<td><strong>L3</strong></td>
<td><strong>L3</strong></td>
</tr>
<tr>
<td><strong>T5</strong></td>
<td><strong>L3</strong></td>
<td><strong>L3</strong></td>
</tr>
</tbody>
</table>

*Note: The table above represents the exemplification, explanation, and engagement in teachers' MDI, focusing on the mediation of the object of learning.
**PD itself**
- Learning study – examples (Pillay)
- Model – object focused PD (Moalosi)

**Teachers/teaching**
- Textbook use (relationship) (Leshota)
- Recontextualising explanation (Luxomo)
- Smk/pck interview scenarios (Patahuddin)
  - Identity (Kambule)

**Learners/learning**
- Functions discourse (Essack)
- Algebra (test items and interviews)
  - Identity (Otulaja)

**Qualitative studies/stories**
Outputs
Development outputs

Model of PD and materials

- Transition Maths Courses (TM1, TM2)
- Maths Teaching Framework in schools
  - 3 week mini lessons/learning study
  - MTF discursive resource (shared language)
- Other materials – integers, functions, ...
- Collaborators and expanding influence

Teachers and teacher educators

- Course completion (50+ teachers)
- Professionalising teachers - AMESA (11 in 2011; 12 in 2013, 2014)
- Human capital development in PD

Schools

- Maths departments

Learners – indirect overall
LMP – direct, pilot
Research outputs

Publications

10 Peer reviewed journal articles
   (key papers now in preparation)
4 papers in professional journal
6 book chapters
1 Book in preparation
13 reviewed conference papers
22 other conference papers
12 keynote presentations

Graduations

Doctoral students (9)
5 passed, 1 under exam, 2 completing, 1 in process

Masters students (10)
6 passed, 2 corrections, 2 completing

Honours projects (20)
15 completed, 5 in process

Postdoctoral fellows (4)
Who?  
What?  
How?
Teacher’s mathematical discourse in instruction (MDI)

• Implicated in, but only a part of a set of practices and conditions that produce poor performance across our schools.

• Significance of ‘talk’ in mathematics pedagogy.

• It matters deeply, how teachers’ mathematical discourse in instruction supports (or not) mathematical learning.
Our starting point on teaching

• Teaching has purpose – there is something to be learned ... **object of learning** (concept, procedure or algorithm, meta-mathematical/practice)

• bringing that into focus is central to the work of teaching

• we privilege the development of scientific concepts, including movement towards objectification in mathematics discourse.
Our intervention - the goal

• We set out to strengthen teachers’ relationship to mathematics, and through this shape their ‘discourse’, firstly in and for themselves, and then in their practice (PD)
  – Not only FET – Grade 9 – 10 critical transition point

• And then to be able describe whether and how this shifts over time, in what ways, and how this is related to what is made available to learn, and to learning gains (RESEARCH)
PD model

Hidden in here - unintended ‘process and outcome’ – training the trainers
• **Two ‘20 day courses’**
  - Critical transitions
    » Transition Maths 1: Gr 9 – 10
    » Transition Maths 2: Gr 11/12 – tertiary education)
  - Focused on mathematics knowledge for teaching – (SMK/pck) - MDI
  - Working on practice – maths teaching framework

• **Reversioned learning/lesson study’**
Key operating principles

• Participation as joint commitment and enterprise of the school, individual teachers and the project (and so the University).
• 20 days – 8 X 2 days at Wits (Release from school on 10 days; 6 days teacher’s time); 4 days equivalent support in school
• Time for teachers to work at their mathematics and teaching over time, and between sessions
• Resources for the school ... supporting ‘successful participation’ of the teachers (funds, technology).
• Potential for ‘spreading out’ - lean and so “cost effective”
Transition Maths courses

Transition Maths 1
• Grade 9/10 teachers
• Maths content: **algebra**, **functions**, geometry and trigonometry
• Teaching content: exemplifying, explaining, learner engagement
• Technology – for mathematising (geogebra), information access and communication

Curve and pipeline ...

More learners better prepared for Grade 10, more teachers available for FET

Transition Maths 2
• Grade 11/12 teachers
• Maths content: **algebra**, **functions**, **calculus**, geometry and trigonometry
• Teaching content: exemplification, explaining, learner engagement.
• Technology

Curve and pipeline ...

More As Bs and Cs. Increase cognitive demand, increasing pace and coverage
In school learning/lesson study with a structuring framework (MTF)

- Studying teaching together (plan, teach ...)
- Using a discursive resource
  - Maths Teaching Framework (MTF)
- Teachers teaching their own learners
- Other teachers observing
- 3-week block; 3 blocks in 2014; ‘curriculum’
- Clusters of schools
# Object of learning: teaching $x$ to $y$

<table>
<thead>
<tr>
<th>Examples and tasks</th>
<th>Explanations and talk</th>
<th>Learner activity</th>
</tr>
</thead>
</table>
| **What examples are used?**  
- To start off the lesson  
- To develop the lesson  
  (these may be “examples of”)  
- To introduce a concept  
- To ask questions  
- To explain further  
- For learners to practise/consolidate  
  (these are “examples for”) |
| **What kinds of explanations are offered?**  
  - What (and why)  
  - How (and why) |
| **What work do learners do?**  
  e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class |
| **What are the associated tasks?**  
  - What are learners required to do with the example/s?  
  - How do these combine to build key concepts and skills? |
| **What representations are used?**  
  ➢ How do these help to build the key concepts and skills? |
| ➢ How does their activity help to build key concepts and skills? |

**Coherence:** Are there coherent connections between the object of learning, examples, tasks and explanations?
## Maths Teaching Framework v2 – Focusing on explanations

### Object of learning

**Explanation**

What does the teacher say and do to help learners make sense of the mathematics beyond the current lesson?

<table>
<thead>
<tr>
<th>What is written?</th>
<th>What is said?</th>
<th>How is the maths justified?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>What does the teacher write (publicly) regarding the mathematical object?</strong></td>
<td><strong>How does the teacher talk about the mathematical object?</strong></td>
<td><strong>How does the teacher justify the mathematics?</strong></td>
</tr>
<tr>
<td>Words, phrases, sentences</td>
<td>Colloquial language</td>
<td>Non-mathematical cues</td>
</tr>
<tr>
<td>Terminology and expressions</td>
<td>Everyday language</td>
<td>Visual cues, mnemonics</td>
</tr>
<tr>
<td>Graphs, illustrations, figures</td>
<td>e.g. “taking x to the other side”</td>
<td>e.g. smiley parabola</td>
</tr>
<tr>
<td>Definitions</td>
<td>Ambiguous referents for objects</td>
<td>Metaphor related to features of real objects</td>
</tr>
<tr>
<td>Procedures</td>
<td>e.g. this, that, thing</td>
<td>e.g. This is how it “looks”, “sounds”, “how you remember”</td>
</tr>
<tr>
<td>Solutions</td>
<td>Some mathematical language to name object, component</td>
<td>Local mathematical</td>
</tr>
<tr>
<td>Proofs</td>
<td>e.g. factor, parabola, derivative</td>
<td>Specific/single cases</td>
</tr>
</tbody>
</table>

- Reading a string of symbols
- e.g. “x into x plus 2”
- Established short-cuts and conventions
- e.g. FOIL, SOHCAHTOA

- Extended and appropriate mathematical language to name mathematical objects and procedures
- e.g. “the product of two binomials”, “subtracting the additive inverse”

**Note:** A general mathematical justification could be partial/incomplete/full.
Week 1

Design lesson
Decide on:
• Mathematical focus
• Examples & tasks
• Learner activity
• Key explanations
• Representations
• Who will teach

Grade 10 linear inequalities
June exam: $-7 < -2x - 5 \leq 9$

Objects of learning
Solve linear inequalities
Represent solution on number line and using interval notation

Key explanation
How to explain: 
$-x > 6$
but $x < -6$

Learner activity
Design card-matching activity linking 3 representations (no. line, interval, symbolic algebraic forms)
<table>
<thead>
<tr>
<th>WMCS Mathematics Teaching Framework</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Object of learning:</strong></td>
</tr>
<tr>
<td><strong>Examples and related tasks</strong></td>
</tr>
<tr>
<td>Identify all examples chosen. How are examples sequenced?</td>
</tr>
<tr>
<td><strong>Explanations</strong></td>
</tr>
<tr>
<td>Do explanations focus on how and/or what? Is attention given to why in explanations? What representations are used?</td>
</tr>
<tr>
<td><strong>Learner activity and comment</strong></td>
</tr>
<tr>
<td>What are learners doing? Engaged with? Note particularly what learners have difficulty with and how this is noticed.</td>
</tr>
</tbody>
</table>
Week 1

Design lesson
Decide on:
• Mathematical focus
• Examples & tasks
• Learner activity
• Key explanations
• Representations
• Who will teach

Week 2

Teach and reflect
• Teacher A teaches lesson to group A
• Other teachers observe
• All reflect on lesson in relation to MTF tool
• Revise aspects of lesson

Objects of learning
Solve linear inequalities
Represent solution on number line and using interval notation

\[\begin{align*}
-7 & \leq -2x - 5 \leq 9 \\
7 + 5 & \leq -2x \leq 9 + 5 \\
\frac{12}{2} & \leq \frac{2x}{2} = \frac{14}{2} \\
-1 & \leq x \leq 7 \\
\end{align*}\]

\[x \in (-1, 7)\]
Week 1
Design lesson
Decide on:
• Mathematical focus
• Examples & tasks
• Learner activity
• Key explanations
• Representations
• Who will teach

Week 2
Teach and reflect
• Teacher A teaches lesson to group A
• Other teachers observe
• All reflect on lesson in relation to MTF tool
• Revise aspects of lesson

Week 3
Teach and reflect
• Teacher B teaches lesson to group B
• Other teachers observe
• All reflect on lesson in relation to MTF tool
• Revise aspects of lesson

Questions to reflect on
What was said?
What was written?
How was it justified?
Did they learn what we intended?
From PD and so working on mathematics and teaching (and discursive resource)
to
Researching teaching (and so analytic device)
Our framing

Teachers’ mathematical discourse in instruction (MDI): A socio-cultural framework for describing and studying/working on mathematics teaching

With discursive (Sfard, 2008) and sociological (Bernstein, 1996) influences; and analytic resources recruited from variation theory (Marton et al, 2004)
<table>
<thead>
<tr>
<th>Exemplification</th>
<th>Explanations</th>
<th>Learner activity</th>
</tr>
</thead>
</table>

**Teachers MDI – mediation towards scientific concepts**

**Object of learning**

Operationalising for research
# Object of learning – mediation towards scientific concepts

<table>
<thead>
<tr>
<th>Examples</th>
<th>Tasks</th>
<th>Naming</th>
<th>Legitimating</th>
<th>Learner Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples provide opportunities within lesson for learners to experience</td>
<td>Level 1 – Carry out known operations and procedures e.g. multiply, factorise, solve equation if these had been taught previously</td>
<td>Level 1NM – Colloquial language including ambiguous referents such as this, that, thing ... to refer to the objects</td>
<td>Level 1NM (Non-Math) Visual: Visual cues or mnemonics Metaphor: Relates to features or characteristics of real objects</td>
<td>Level 1 – Learners answer yes/no questions or offer single words to teacher’s unfinished sentence</td>
</tr>
<tr>
<td>Level 1- contrast or generalization</td>
<td>Level 2 – Apply level 1 skills; learners have to decide on (explain choice of) operation and/or procedure to use e.g. Compare/ match representations, classify, ...; also includes tasks about the current lesson</td>
<td>Level 2M – Some mathematics language to name individual objects, components or simply read string of symbols when explaining</td>
<td>Level 2M (Math) (Local) Specific/single case (real-life application or purely mathematical) Established shortcuts; conventions</td>
<td>Level 2 – Learners answer (what/how) questions in phrases/sentences</td>
</tr>
<tr>
<td>Level 2- contrast and generalization</td>
<td>Level 3 – Multiple concepts and connections. e.g. Solve problems in different ways; use multiple representations; pose/construct problems; prove disprove; explain reasoning, etc</td>
<td>Level 3M- Uses appropriate mathematical language</td>
<td>Level 3M (General, partial) equivalent representations, definitions, previously established generalization but explanation unclear or incomplete, principles, structures, properties but unclear/partial</td>
<td>Level 3- Learners answer why questions; present ideas in/for discussion</td>
</tr>
<tr>
<td>Level 3- fusion</td>
<td></td>
<td>Level 4M – Uses appropriate mathematical language in an objectified form.</td>
<td>Level 4M (General, full)</td>
<td></td>
</tr>
</tbody>
</table>
A lesson and its analysis

• Ms O – Grade 10 revision lesson begins with revision of algebraic fractions leading to new work on division of algebraic fractions

• The lesson consists of five events, with a new event marked by a shift in ‘focus’.
  – 1 - review of the meaning of a term in an algebraic expression – 6 varying examples
  – 2 – review of a common factor using just one example of a binomial expression.
  – 3  - new work - four examples (sub-events) of algebraic fractions. The task was simplifying (through factorization) the expressions in each of the numerator and denominator to produce a single term. Complexity increased in terms of the type of factorisation required in successive examples.
  – 4 and 5  - was division of algebraic fractions (positive and negative).

• We work first within an event, to analyse exemplification, explanation and interaction, and then look across events for accumulating ‘mathematical coherence’ and ‘mediation towards the scientific’

• Illustrate with event 4, and detail with 4.3
Event 4: Sub-events 4.1 - 4.4

Examples and tasks

T writes example 4.1 on the board, asks questions mainly requiring yes/no answers, completion of sentences by learners in unison, leading to the solution. Occasionally learners respond with a phrase or sentence to a what or how question. Any why question she answers herself. Examples 4.2 and 4.3 follow the same form. The transcript extract below details the talk leading to the solution for 4.3. Example 4.4 is then given for learners to do independently.

\[
\begin{align*}
4.1 & \quad 4.2 & \quad 4.3 & \quad 4.4 \\
\frac{2}{6} \div \frac{2}{3} & \quad \frac{2x}{6x} \div \frac{2x}{3x} & \quad \frac{x^2-x^2}{4} \div \frac{x^2}{8} & \quad \frac{x^2-x}{x^2+x-2} \div \frac{x^2+4x}{x^2-4} \times \frac{3x+12}{1}
\end{align*}
\]

Examples: Level 3 - Variation is by generalization and fusion.

4.1-4.3 The structure of the division of one fraction by another is kept constant and terms varied. These range from simple to complex; from numerical to algebraic. 4.4 extends to three fractions and a product (Generalization).

Egs 4.3 and 4.4 require associating common factor with fraction division (Fusion).

Tasks: Level 1 - Perform the indicated operations to simplify expressions
### Sub-Event 4.3  
**Talk and legitimating criteria**

Analysis of explanatory talk highlighted as follows: *italics for colloquial* and *underlining for formal language*; and *bold type for criteria/legitimations*;

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. T: It’s <em>one and the same thing</em>. They give you <em>something like this</em> <em>(writes symbols on board).... x cubed minus x squared the whole thing over, over four .... divided by x squared over eight...ok?</em></td>
<td></td>
</tr>
<tr>
<td>2. Ls: Yes</td>
<td></td>
</tr>
<tr>
<td>3. T: So it’s, it’s <em>one and the same</em> concept. <em>Over here</em> <em>(points to number 4.1 ) you just have two numbers, a fraction divided by a fraction, ok?</em></td>
<td></td>
</tr>
<tr>
<td>Ls: Yes</td>
<td></td>
</tr>
<tr>
<td>4. T: Over here <em>(pointing back to 4.3) is the same thing. I’ve got, here’s one fraction divided by one fraction (circles each fraction). So the examiner is just making your life difficult, ok?</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>5. T: So....what are we going to do <em>over here?</em> <em>(points to first fraction)</em></td>
<td></td>
</tr>
<tr>
<td>6. Ls (some): we are going to divide</td>
<td></td>
</tr>
<tr>
<td>7. T: <em>...remember the rule</em> that we learnt <em>over there?</em> <em>(points to similar expression, Event 2,factors obtained to simplify fraction)</em></td>
<td></td>
</tr>
<tr>
<td>8. Ls: Yes</td>
<td></td>
</tr>
<tr>
<td>9. T: For before we can go and divide, <strong>what must I do?</strong></td>
<td></td>
</tr>
<tr>
<td>10.Ls: <em>Take out</em> the common factor.</td>
<td></td>
</tr>
<tr>
<td>13. T: So, the same thing applies here. <em>It is everything that you, that you have learned, but they just put it into one thing to make it look a bit complicated. It’s actually very simple...ok?</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>37. T: <strong>So, you just apply the same principle, it’s just that when it looks complicated just pause and say what must I do here?</strong> Because I know terms <em>like this</em> <em>(points to ,)</em></td>
<td></td>
</tr>
<tr>
<td>I cannot just...go and say <strong>this</strong> <em>(pointing to   divided by this</em> <em>(points to 4) ...ok?</em></td>
<td></td>
</tr>
</tbody>
</table>
Talk: Level 2 – Moves between colloquial talk and some math language (e.g. ln 3) to name individual components or simply read string of symbols when explaining

Legitimation: Level 1 Reference to visual features (e.g. ln 3, 4, 13) and Level 2M (Local) Established shortcuts; conventions (e.g. lns 7, 10, 11, 30) and Level 3M (General) Makes reference to structure/principle but not clear due to naming (e.g. ln 37)

Event 4: Interaction pattern

Interaction pattern: Dominantly Level 1 Ls answer yes/no questions or supply words to T’s unfinished sentence; Occasional Level 2 Ls answer what/how questions in phrases or sentences
Our analysis of Event 4 shows the Teacher operating at:

- **Examples - Level 3**  shift from Level 1

- **Tasks - Level 1** - which remain at the level of learners carrying out known procedures

- **Interaction - Level 1** - learners answers yes, no questions, and provide words/phrases in response to teachers questions on what to do

- **Naming/talk - Level 2** - the teacher’s words while frequently including ambiguous referents, move on to rephrase using mathematical language to name mathematical signifiers and processes shift in movement between

- **Legitimating Criteria** - shift between emphasis on visual features of expression, conventions, with some reference to structure and generality and so across levels 1 - 3 shifts to more movement between
Teacher 3 – township school

### 2012

<table>
<thead>
<tr>
<th>OoL: Simplifying exponential expressions</th>
<th>Examples</th>
<th>Tasks</th>
<th>Naming</th>
<th>Legitimations</th>
<th>Learner Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>H: Laws of Exponents</td>
<td>L1</td>
<td>L1</td>
<td>L3</td>
<td>NA</td>
<td>L1</td>
</tr>
<tr>
<td>Simplifying expo. expressions</td>
<td>L3</td>
<td>L2-L1</td>
<td>L3</td>
<td>L3</td>
<td>L2</td>
</tr>
<tr>
<td>Practice: Simplifying. expo.</td>
<td>NA</td>
<td>L2-L1</td>
<td>L2</td>
<td>L2</td>
<td>L2</td>
</tr>
</tbody>
</table>

### 2013

<table>
<thead>
<tr>
<th>OoL: Simplifying Algebraic Fractions</th>
<th>Examples</th>
<th>Tasks</th>
<th>Naming</th>
<th>Legitimations</th>
<th>Learner Participation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defn. term and factor</td>
<td>L3</td>
<td>L1</td>
<td>L3</td>
<td>L3</td>
<td>L1</td>
</tr>
<tr>
<td>Simplifying alg. fraction</td>
<td>L2</td>
<td>L2-L1</td>
<td>L2</td>
<td>L3</td>
<td>L1</td>
</tr>
<tr>
<td>Division of alg. fraction</td>
<td>L3</td>
<td>L2-L1</td>
<td>L2</td>
<td>L3</td>
<td>L1</td>
</tr>
<tr>
<td>Equiv. expression with neg. expo.</td>
<td>L2</td>
<td>L2-L1</td>
<td>L2</td>
<td>L2</td>
<td>L1</td>
</tr>
</tbody>
</table>
### MDI: Summary

<table>
<thead>
<tr>
<th>Examples</th>
<th>Tasks</th>
<th>Naming</th>
<th>Legitimation</th>
<th>Lear. Part.</th>
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<tr>
<td>Kekana</td>
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<td>L2-L1 L2-L1</td>
<td>L2 L4</td>
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</tbody>
</table>
• The MDI framework is thus helpful in directing work with the teacher (teaching), and in illuminating take up of aspects of MDI within and across teachers (research)

• The MDI framework provides for responsive and responsible description.

• Illustrated MDI on what many would refer to as a ‘traditional’ pedagogy. MDI works as well to describe lessons structured by more open tasks, indeed across ranging practices observed.
• We set out to strengthen secondary teachers’ relationship to mathematics, and through this shape their ‘discourse’, firstly in and for themselves, and then in their practice (PD)

• And then to be able describe whether and how this shifts over time, in what ways, and how related to what is made available to learn, and to learning gains (RESEARCH)
Closing comments

• Progress?

• Constraints?

• Contributions?

• And enormous privilege 😊
Thank you