Local Evidence-Driven Improvement of Mathematics Teaching And Learning Initiative

Progress report of the activities of the UWC FRF Chair in Mathematics Education

July 2013 – June 2014

Cyril Julie
Some of the most familiar images of schooling in South Africa
DEVELOPING TEACHERS’ MATHEMATICAL CONTENT KNOWLEDGE

Teachers have “knowledge gaps”.

Not particular to teachers. They also manifest themselves in the work of highly adept mathematicians and other workers and users of mathematics.

Some tactics and strategies used by adept mathematical workers:
• re-studying/reviewing of ideas that got ‘rustied’ though an extended period of non-use
• discussion of their dilemmas with others and developing clarity through these discussions
• asking their peers for explanations
• self-studying of the mathematics they know they do not command sufficiently
• attending courses/seminars dealing with the topics for which they want to enhance their understanding (in order to use it).
Knowledge gaps of mathematicians and other mathematical workers surface when they are engaged with problems in their practice and in order to address these gaps they resort to the strategies and tactics referred to above

The knowledge gaps are normally self-acknowledged

Teachers’ engagement with mathematics is of a different kind: its teaching and the design and development of various objects pertinent to their teaching of mathematics.

Selection of mathematical activities

Design of mathematical activities

Examination-setting

School mathematically-driven practices are exploited to develop teachers’ knowledge.
Engagement with school mathematics topics during workshops and institutes to enhance teachers’ mathematicalness
On a curious Property of vulgar Fractions.

By Mr. J. Farey, Sen. To Mr. Tilloch

Sir. - On examining lately, some very curious and elaborate Tables of "Complete decimal Quotients," calculated by Henry Goodwyn, Esq. of Blackheath...

If all the possible vulgar fractions of different values, whose greatest denominator (when in their lowest terms) does not exceed any given number, be arranged in the order of their values, or quotients; then if both the numerator and the denominator of any fraction therein, be added to the numerator and the denominator, respectively, of the fraction next but one to it (on either side), the sums will give the fraction next to it; although, perhaps, not in its lowest terms.

For example, if 5 be the greatest denominator given; then are all the possible fractions, when arranged, 1/5, 1/4, 1/3, 2/5, 1/2, 3/5, 2/3, 3/4, and 4/5; taking 1/3, as the given fraction, we have (1+1)/(5+3) = 2/8 = 1/4 the next smaller fraction than 1/3; or (1+1)/(3+2) = 2/5, the next larger fraction to 1/3...

I am not acquainted, whether this curious property of vulgar fractions has been before pointed out?; or whether it may admit of any easy or general demonstration?; which are points on which I should be glad to learn the sentiments of some of your mathematical readers; and am

Sir,
Your obedient humble servant,

J. Farey.
What sense do you make of it? To what common ‘error’ related to the addition of fractions does it refer to? Generate examples, different to the ones given in the article.

Ascending order

Descending order
A ‘counter-example’

Puzzling!
Attend the conditions for the validity of the series

arranged in the order of their values

closing comments: careful reading of mathematical text
Farey series, prime numbers and the unsolved Riemann hypothesis
Take a fraction. Subtract the top from the bottom and put it on the top of a new fraction; then add the top and bottom and put it at the bottom of the new fraction. Repeat the same operation. What do you notice? Does it always work?

Hypothesis: if one continues with the process you get a multiple of the fraction you start with, starting with 2 and moving up in powers of 2
Generalised ‘explanation’ of the hypothesis

\[
\frac{d}{b} \quad \frac{b-a}{2a} \quad \frac{2}{1b} \quad \frac{2(b-a)}{2(a+b)} \quad \frac{4a}{4b}
\]
Geometry
Requested by reachers requested

Re-insertion in the Further Education and Training band

Examined for the first time the National Senior Certificate examination for Mathematics in 2014 examinations.

<table>
<thead>
<tr>
<th>Session 1</th>
<th>Rider strategies</th>
<th>Congruency Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Through engaging with grade 11 geometry and geometry done in earlier grades</td>
<td>Direct Application of theorems(s)</td>
</tr>
<tr>
<td></td>
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<td>The algebraic approach &amp; its advantages</td>
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<tr>
<td></td>
<td></td>
<td>Use other branches (topics) of mathematics</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Reductio ad absurdum</td>
</tr>
<tr>
<td>Session 3</td>
<td>Similarity</td>
<td>1. Approach to teaching Similarity</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2. Riders</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(a) Calculation type: Simple → moderate → difficult</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b) Proving type: Simple → moderate → difficult</td>
</tr>
</tbody>
</table>
Phase groups—grades 8 and 9 and Grade 10 to 12

General strategy to approach geometry riders was presented to all participants

Approach to Solving/Proving Rider

- Read Question
- Underline key words and given statements
- Insert what is onto diagram
- Analyse Problem
- Chain of thought
- Chosen strategy
- Implement strategy
- Evaluate your answer to see if it makes sense
Grade 8 & 9 teachers the construction of a square with a straight edge and a compass.

Teachers’ request to deal with constructions with “geometry sets” since “it was in the CAPS document”.

Observation

Initial cautiousness and many attempts to obtain an acceptable figure with the instruments at hand
Constructions with a straight edge and compass played a significant part in the historical development of Mathematics.

Enlarge teachers’ repertoire of ‘stories’ to relay to learners the roles of tools in the development of Mathematics.
Teacher-generated discussions on solutions presented to geometry problems

In what order should triangles be named in presented solutions for congruency for triangles?

Should be named and written such that the equal sides and angles could easily be identified.
2013 ANA problem

In the figure below $\angle ABD = \angle CDB = 90^\circ$ and $AD = BC$.

Presented solution

Prove that $\triangle ABD \cong \triangle CDB$.

Should angles not be included by the two sides?

Not necessary for right-angle triangles
FET phase

Discussion of the “Mathematics DBE/2014 Examination Guidelines: Euclidean geometry & measurement”

Engagement with school-like geometry problems

ABC is any triangle. E and F are midpoints of AC and AB. BE is joined and produced to G such that EG = BE. CF is joined and produced to H such that FH = CF. Prove that the points H, A, and G are collinear.
HA//BC through the congruency of Δ’s HAF and CBF. This led to the conclusion that $HAF = x$

Established that $B\hat{A}C = 180^\circ - (x + y)$. $G\hat{A}E = y$ was calculated by using $180^\circ - x - (180^\circ - (x + y))$. AG//BF was claimed due to the equality of the angles, $G\hat{A}E$ and $B\hat{C}E$. Through the use of transitivity, HA//BC and AG//BC, it was concluded that HAG is a straight line.
Flaw—using what has to be proved as an element of in the proof

Two alternate proofs were presented.

The midpoint theorem was used to establish $BE = EG$ and establishing $FE//HA$ and $FE//AG$.

The bisection of the diagonals of a parallelogram to establish $HA//BC$ and $AG//BC$ from which the colinearity was deduced.
Misunderstandings learners develop due to a lack of focus on the saliency of concepts and a hastiness to do riders.

“Angle subtended by chord of a circle”.

generally taught when the accompanying theorem is taught and its use in riders in circle geometry, the generality is lost and all angles subtended by chords are deemed to be at the circumference.

Thinking:

Conceptual understanding

Understanding concepts
Proofs by contradiction, the application of the contradiction rule was dealt with through the solution of logical puzzles

Problem used by logician Raymond Smullyan

Knights—always tell the truth
Knaves—always lie

Two natives A and B address you as follows:
A says: Both of us are knights.
B says: A is a knave.
What are A and B?

Circular nature of arguments, as an instance of the difficulty of proving by contradiction
Appropriateness of Activity for My Practice

Response

- Very High (4)
- High (3)
- Low (2)
- Very Low (1)
- Open (99)

Fractions
Geometry
Date: 2014/07/12 01:03 PM
Subject: Outstanding workshop

Sir

AMESA is one of the most powerful congresses for Maths educators in particular, one comes out a different individual when it comes to development and cognitive growth in general. I was part of your Euclidean Geometry session where I had a chance to shine also with one of the interesting problems u gave us.

…This is for any chances you may have for me for growth and any opportunity of development that may inspire a Maths educator who dreams high like me.

One day I wish I could be at your level, you inspired me a lot

Emerging Claim:

The way the project deals with mathematical knowledge is responsive to teachers’ needs for enhancing their school mathematics subject matter knowledge.
Elementary mathematical modelling

Investigations, project work, tasks are very sterile and mundane

Short course (16 hours) on elementary mathematical modelling

Apply for attendance

Content:

The notion of a mathematical model of a “real-life” situation

the mathematical modelling cycle and associated competencies

kinds of mathematical models

developing mathematical model

mathematical modelling in the school mathematics curriculum
The current rankings of the 2012 London Olympics, published by most of the international media, rank countries by order of gold medals won. This is clearly silly, since it ignores silver and bronze medals entirely. Another popular ranking system published by the media is to tally the total number of medals won. This is also patently simplistic.

More than a hundred years ago, the British press devised a much fairer ranking system: each gold is worth 5, each silver is worth 3 and each bronze is worth 1. So a little bit of multiplication and addition is necessary.

Here are my calculations of the Weighted Ranks for the London Olympics. There are some surprises: first of all Australia places 8th, not 10th; Great Britain actually is out of the minor placings, beaten into third place by Russia – perhaps this is the reason the British press don’t seem to be too keen on promoting this fairer system that they actually invented! – Spain goes from 21st to 14th, and Canada goes from 36th place to 22nd.

Congratulations to all our athletes for their excellent 8th place in a highly competitive meet. Here are the revised weighted rankings, based on the equation Weighted Total = 5G + 3S + B.

Associate Professor Norman Wildberger
Link to indexes as general models

Competencies:
  • declaration of assumptions
  • building a model based on assumptions
  • testing and validating of a model
  • possible improvement of the model

What space is occupied by the tree?
Initial discussions:

meaning of ‘space’

a two- or three-dimensional situation should be considered
Ranking of universities based on research output by considering academic staff complements, student demographics and ‘financial strength’ of universities

Optimization of parking space for vehicles in a corner rectangular parking lot
The control of the elephant herd in the Kruger National Park given relocation data based on age and gender for 1998 and 1999.

* The herd dynamics within the group.
* Food resources

**Model**

- The two independent variables
  * Number of babies
  * Number of deaths
- The two dependent variables
  * Time
  * Population

Creating a formula

As time increases, more babies will be born and more elephants will die.

Ideal situation

\[ \Delta t = \text{babies} - \text{death} + \text{f} \]

\[ t_1 = b_i - d_i \]

\[ t_2 = (b_i - d_i) + (b_2 - d_2) \]

\[ N(t) = N(b_i - d_i) \]

**NB:**  
- \( b = \text{babies} \)
- \( d = \text{death} \)

**Excel Table:**

<table>
<thead>
<tr>
<th>Years</th>
<th>Total Population</th>
<th>1998</th>
<th>1999</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Numer that will die +/- 25%</td>
<td>Bulls and Cows (1:1)</td>
<td>% That will conceive</td>
</tr>
<tr>
<td>1997</td>
<td>2750</td>
<td>4125</td>
<td>1375</td>
</tr>
<tr>
<td>1998</td>
<td>4125</td>
<td>4125</td>
<td>1375</td>
</tr>
</tbody>
</table>
WHAT SPACE DOES THE TREE OCCUPY?

ASSUMPTIONS
- The radius of the trunk is constant
- Visibility part used only
- No space between leaves and branches
- Trunk – cylindrical
- Top of tree conical
- Trunk is measured from bottom to first branch of the tree

MODEL
The volume of the tree = the volume of cone + volume of trunk

\[ \frac{1}{3} \pi R^2 h + \pi r^2 h = \pi \left( \frac{1}{3} R^2 H + r^2 h \right) \]
An invited visiting academic, Prof Barath Sririman from the University of Montana, acted as external respondent for the presentations of the models. He also commented on the presentation of the final “tree model”. The final model is given below. In particular his comments centred around the generalisibility of the model. One of the teachers worked on a generalisation which he presented at a subsequent general institute.

SPACE OCCUPIED BY ANY NATURAL OBJECT =
THE VOLUME OF AN ASSUMED SHAPE OF THAT OBJECT OR THE SUM OF THE ASSUMED SHAPES OF THAT OBJECTS
“This weekend found us looking at a mountain on the Friday that we arrived. That mountain was broken down the next morning and modelling became easier as we understood it better now...I am glad that I came to this workshop. It broadened my knowledge base...”

“...when I arrived on Friday for the course I was not sure what to expect. Even when the first scenario were given I still was not sure I was still a bit worried that I could not do it...As the weekend progressed however what the concept of what mathematical modeling is became much clearer...I now have a better understanding of mathematical modelling and how to use it in my classroom.”

Onderwysers het ook gesukkel om die model bymekaar te sit. To me that was an eye opener. Somtyds veronderstel ons net dat die leerders die werk kan doen... ‘Real life’ situations is now so real to me. Ek het ‘n “wow”moment gehad die naweeek.”

“Mathematical modelling is very interesting and yet confusing when it is the first time you are introduced to it. Its interesting that we keep on saying there is ‘Maths’ everywhere and yet when we had to apply it we find it difficult.
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WORKSHOPS

Didactical issues

quarterly planning

lesson planning

design of teaching

examination issues
When questions are turned around, learners find them difficult to deal with

1.4  Given the expression $\frac{x-y}{3} + 4 - x^2$

Circle the letter of the incorrect statement

A  The expression consists of 3 terms
B  The coefficient of $x$ is 1
C  The coefficient of $x^2$ is -1
D  The expression contains 2 variables
A SCHEME OF MATHEMATICAL QUESTION TYPES

Five different types

Standard/direct questions

Non-standard questions to convert to standard form questions

Reversal questions

Evaluative questions

Learner constructed questions.
Reversal question:

Find a quadratic equation of which one root is irrational and the other a negative integer

Evaluative question

For the question:

Simplify without the use of a calculator: \[
\frac{\sin 104^\circ(2\cos^2 15^\circ - 1)}{\tan 38^\circ \cdot \sin^2 412^\circ}
\]

A learner produced the following:

\[
\begin{align*}
= \sin 104^\circ(2\cos^2 15^\circ - 1) \\
\quad \tan 38^\circ \cdot \sin^2 412^\circ \\
= \sin (90^\circ + 14^\circ) \cdot 2 \cos^2 14^\circ \\
\quad \tan (90^\circ - 52^\circ) \cdot \sin^2 (360^\circ + 52^\circ) \\
= \sin 14^\circ \cdot 2 \cos^2 14^\circ \\
\quad \tan 52^\circ \cdot \sin 52^\circ \\
\end{align*}
\]

Indicate with reasons where the learner went wrong
A teacher report on implementation

After I did the sum and difference of cubes...I mean one of the exercises in the textbook has a lot of questions on the sum and difference of cubes. I went through each one and told them to analyse it. And...once we’ve factorize it, I told them look at...in your short bracket, look at your first term and in your long bracket, which was that one [points to the bracket containing the trinomial in the figure above] look at the first term of the long bracket and what is the relationship in each and every one of them. And then they said “Oh...That is that [pointing to \(x^2\)] is that one [pointing \(x\)] squared.” So I said that is what it’s gonna be in every single case.

And then I told them “Look at the middle term [whilst pointing to \(bx\)] and try to find what is the relationship between these two numbers [shifting the pointing to the 2 in the first bracket]? They went through and they said “OK. It’s the same number with the same digits but the signs are different” ...So I told them “That is what it is gonna be in every single case... And I told them to look at this one [points to the 2 in the ‘small’ bracket] and this one [points to the \(c\) in the ‘long’ bracket] and then they said “Oh, that’s the square again.” Then I told them to look at what they starting with [points to \(k\) on the right-hand side] and look at the first bracket [pointing to the 2] and they said “Ok, Miss, it like that thingie with the three on it for the first number and the second number.” So that is how I taught it to them.
Some teachers are beginning to appropriate the scheme and using it, albeit in limited instances, in their practice.
One of the strategies developed to address the issue of forgetting and learners not consolidating their knowledge through independent work was revision—in-class practising of previously covered work.

Incremental rehearsal, distributed practice, ‘snappies’

Grade 9   Spiral Revision Exercizes   Card 39
Real Numbers; Linear and Exponential Equations

1. Learners were asked to calculate the value of $\sqrt[3]{64}$, without using a calculator. Learner A’s answer was 4 and learner B’s answer was 8. Which learner is correct? Show through calculations how the correct answer was arrived at.

2. Solve for $x$: $x^2 - 25 = 0$

3. Solve for $x$: $8^x = 32$
Collaborative setting of examinations and the development of items

Simplify \( \frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4} \)

A possible scaffolded reformulation offered was “(a) State why \(-\frac{x+2}{2} = +\frac{-x-2}{2}\) is true and (b) Simplify \( \frac{2x+1}{4} - \frac{x+2}{2} - \frac{1}{4} \).
(Re-) discussion on issues of concern

Discuss in your groups factors and/or issues which work against you implementing the teaching strategies suggested by the project and list the 3 that your group feel are the most pressing

Impinging directly on classroom teaching

(1) Lazy learners
(2) Large classes
(3) Teacher overload
(4) Overcrowded syllabus
(5) Discipline problems in class
(6) Infringements on teaching time
Motivation in the private sector

Director of executive education at the Wits Business School

the soft skills are becoming more important in the modern world of business and that professionals do more become responsible for “motivating employees.”

these skills [of motivating people are not] “necessarily gained en route.”
Contingency meeting

Prep for exam
- Extra classes for revision
- Prep workbooks and practice exams
- Surprised test
- 7 days revision before the exam
- Answer questions
- Spirial revision

CURRICULUM PACING STRATEGIES
1. Regular subject meetings (weekly)
2. Tutorials (monthly) - work towards content in tut.
3. Common lesson plans
4. LITNUM tests (every second week)
5. Discussion of what would be covered in tests etc. (who sets the paper)

# TO LEAVE OUT CERTAIN TOPICS
- THAT ARE NOT ESSENTIAL IN THE FET. I.E. TRANSFORMATION GEOMETRY
- CONSTRUCTION OF GEOMETRIC FIGURES
- SURFACE AREA OF 3-D SHAPES
- VOLUME EXCEPT FO R PRISM
- OR THE ABOVE MENTIONED TOPICS CAN BE DONE LAST IF THERE'S TIME.
Goals for 2014

Long term: (years)
  Consolidation: confidence in the knowledge of the Math content.
  Be part of the successful implementation of the new Ledimtali model.
  Development of a math community (interaction)

Medium Term: Months
  Increase the pass rate & quality of grades/codes of learners
  Psychology issue (like) cultivate a love for maths.
  Conducive environment

Short term: (weeks/days)
  Make more learners understand the complex concepts e.g. trigs, geometry & calculus
  Enrol in higher institutions of learning
  ID gaps & remedy incorrect answers
  Solid foundation
Own personal goals for 2014

Do spiral revision

More independent Iwork learners in class work meaning “

Increasing the pass rate
“Black box” related to the NSC Mathematics

CRITERION: 1  TECHNICAL CRITERIA

- Cover page – all relevant details
- Instructions – clearly specified and unambiguous.
- Lay out – candidate friendly.
- Correct numbering.
- Appropriate fonts
- Mark allocations clearly indicated – same as that on the memo
- Quality of illustrations, graphs, tables etc – appropriate and print ready.

CRITERION: 6  PREDICTABILITY

- Questions are of such a nature that they cannot be easily spotted or predicted.
- There is no repetition of questions from the past three years’ question papers.
- The paper contains an appropriate degree of innovation.
CRITERION: 3 COGNITIVE SKILLS

• Appropriate distribution in terms of cognitive levels (Bloom’s taxonomy or any other taxonomy that may have been used).

• Representative of the latest developments in the teaching of this knowledge field.

• Scaffolded in terms of degree of challenge i.e. easy questions leading into more challenging questions - also within questions.

• Opportunities to assess reasoning; ability to communicate; ability to translate from verbal to symbolic; ability to compare and contrast; ability to see causal relationship; ability to express an argument clearly.

Examples: Knowledge

• Write down the next three terms in the sequence: 103; 105; 107.... [Easy]

• Determine the factors of 64 [Moderate]

• Write down the prime numbers that are factors of 36. [Difficult]
Teachers generally attached a high value to institutes for providing them with the opportunity to meet and share ideas about Mathematics and its teaching.
CLASSROOM SUPPORT

Fieldworkers visit classrooms 3 days a week to support teachers with the implementation of the designed teaching model; presenting exemplary lessons; assist with planning lessons and present reflective feedback to teachers on their teaching pointing out areas for possible improvement particularly with respect to content elements.

PROGRESS REPORT OF THE ACTIVITIES BY FIED WORKER YYY FOR THE WEEK-ENDING 20 MARCH 2014

<table>
<thead>
<tr>
<th>SCHOOL: XXX</th>
<th>SUPPORTED EDUCATOR: ZZZ</th>
<th>GRADE: ???</th>
</tr>
</thead>
<tbody>
<tr>
<td>LESSON TOPIC: NUMBER PATTERNS</td>
<td>PRESENTER(S): YYY/ZZZ</td>
<td></td>
</tr>
</tbody>
</table>

Post-lesson talks with teacher
EVALUATION MATTERS

8(d): Institutes--mathematical problems

5(a): workshops--teaching strategies based on learners' performance

5(c): workshops--designing 'spiral revision' activities

5(b): workshops--lessons based on "Intentional teaching"

$r_s = .580^{**}$

$CPD$ Effectiveness

$r_s = .478$

$r_s = .520^{*}$

$r_s = .820^{**}$
38% increase in the number of candidates who sat for the 2013 NSC Mathematics examination relative to 2012

General decrease in the quality of the passes
Potential threat

idolize, criticized and victimize

Inverted U - curve
how can interest be maintained to move the project towards cooperation—collaboration range
Local Evidence-Driven Improvement of Mathematics Teaching And Learning Initiative

THANK YOU